

# The effect of synchronized area on SOC behavior in a kind of Neural Network Model \*

Xiao Wei Zhao and Tian Lun Chen<sup>†</sup>

*Department of Physics, Nankai University, Tianjin 300071, P.R.China*

(Dated:)

## Abstract

Based on the LISSOM model and the OFC earthquake model, we introduce a self-organized feature map Neural Network model . It displays a "Self Organized Criticality"(SOC) behavior. It can be seen that the feature area (synchronized area)produced by self-organized process brings about some definite effect on SOC behavior and the system evolves into a "partly-synchronized" state. For explaining this phenomena , a quasi-OFC earthquake model is simulated.

---

\* The project supported by P.R.China National Basic Research Project "Nonlinear Science"

<sup>†</sup>Electronic address: xwzhao@263.net chenzh@nankai.edu.cn

## I. INTRODUCTION

A few years ago, Bak, Tang and Wiesenfeld introduced the concept of the "Self-Organized Criticality" (SOC) in sand pile model [1]. From then on, this concept has been widely studied in some extended dissipative dynamical systems, such as earthquake [2], biology evolution [3], and so on. It is shown that all these systems can naturally evolve into a "critical state" with no intrinsic spatial and temporal scales through a self-organized process without the need to fine-tune parameters of the system. This critical state is characterized by a power-law distribution of avalanche sizes, where the size is the total number of toppling events or unstable units.

Now, the research on SOC has come into a new level. One has studied several factors' influence on SOC behavior, such as network size, periodic or nonperiodic boundary conditions, local dynamics variable is conservative or not, and so on. Many investigators believe that it is intrinsic-stability(order) and variability(disorder)'s common action make the system evolves into a "frozen disorder" (SOC) state [4, 5, 6]. It is the combination feature of stability and variability, and it's complex spatial-temporal dynamic behavior, make the system in SOC state have maximum complexity and latent computing potency.

The brain is a complex system and its information process has the properties of stability and variability – on one hand, there are relative stable information stored mechanism, and stored area in brain(such as feature area in cortex) ; on the other hand, the brain is influenced by the environment and one should continuous update knowledge and concepts. The similarity between the SOC systems and the brain has lead us to study Artifical Neural Network(ANN) and SOC together. There is some SOC behavior shown in the neuron network model introduced by our group [7].

The brain is also a complex system with highly complexity, highly order and special structure. The structure must have the effect on the brain's dynamics behavior. Our neuron network can also produce some special structure, so we believe that the SOC behavior shown in our model must have it's own special behavior and rule.

In this paper, the feature area(produced by self-organized process)'s definite effect on SOC behavior has been studied.

## II. MODEL

Here we propose a two-dimensional neural network model of square lattice. This model is a kind of serial self-organized neural network model, based on the LISSOM model [8]. When a  $h$ -dimensional vector  $\zeta$  is inputted, the state  $\eta_{ij}(t)$  of the neuron  $(i, j)$  at time  $t$  is changed according to the formula:

$$\begin{aligned}\eta_{ij} &= \sigma \left\{ \sum_h \mu_{ij,h} \zeta_h + \gamma_e \sum_{kl} E_{ij,kl} \eta_{kl}(t-1) \right. \\ &\quad \left. - \gamma_i \sum_{k'l'} I_{ij,k'l'} \eta_{k'l'}(t-1) \right\} \\ &= \sigma \{ f_{ij}(t) \}\end{aligned}\quad (1)$$

where  $\sigma(x)$  is an active function, and we design it as sign function, i.e., if  $x \geq 0$ , then  $\sigma(x) = 1$ , otherwise  $\sigma(x) = -1$ .  $\mu_{ij,h}$  is an afferent input weight vector;  $E_{ij,kl}$  is the excitatory lateral connection weight on the connection from the neuron  $(k, l)$  to the  $(i, j)$  neuron;  $I_{ij,kl}$  is the inhibitory connection weight.  $f_{ij}(t)$  is the local field of the neuron  $(i, j)$  at time  $t$ .  $\gamma_e$  and  $\gamma_i$  are constant factors. The adjustment of those three connection weights is as following according to the dynamic Hebb rule:

$$\begin{aligned}\mu_{ij,h}(t+1) &= \frac{\mu_{ij,h}(t) + \alpha \eta_{ij} \xi_h}{\{\sum_h [\mu_{ij,h}(t) + \alpha \eta_{ij} \xi_h]^2\}^{1/2}}, \\ E_{ij,kl}(t+1) &= \frac{E_{ij,kl}(t) + \alpha_E \eta_{ij} \eta_{kl}}{\sum_{ij} [E_{ij,kl}(t) + \alpha_E \eta_{ij} \eta_{kl}]}, \\ I_{ij,k'l'}(t+1) &= \frac{I_{ij,k'l'}(t) + \alpha_I \eta_{ij} \eta_{k'l'}}{\sum_{k'l'} [I_{ij,k'l'}(t) + \alpha_I \eta_{ij} \eta_{k'l'}]}. \end{aligned}\quad (2)$$

where  $\alpha, \alpha_E, \alpha_I$  are the learning rates.

Note the change of the neuron states is quick and the adjustment of the connection weights is slow. Usually, after over 10 iterations of the neuron states when any pattern is inputted (at this time, the network state becomes an attractor in the state space, often a fixed point.), all connection weights are updated once. Thus, after learning a while, the lateral connection weight self-evolves into the "Mexican hat" profile, the afferent input weight self-organizes into a topological map of the input space, the neuron network can produce some special feature areas, and the state of the neural network is evolved from disordered case to stable and topological case in state space.

Then we introduce the following interactive process between the neurons, similar with the pulse coupled interaction:

- 1) When the neuron  $(i, j)$  is stable, i.e.,  $\eta_{ij}(t) = \sigma\{f_{ij}(t - 1)\}$ , it doesn't influence the others;
- 2) If the neuron  $(i, j)$  is unstable, i.e.,  $\eta_{ij} \neq \sigma\{f_{ij}(t - 1)\}$  or  $f_{ij}(t - 1) = 0$ , then the nearest neighbors  $(i', j')$  around this unstable neuron will receive a pulse respectively and their local fields will be changed. At the same time, the neuron  $(i, j)$  becomes stable again, depending on the formula (1);
- 3) When all neurons of the neural network are stable, we choose the minimum  $g$  among the absolute values of all local fields  $f_{ij}$  and drive the local field of every neuron, i.e.,

$$f_{ij} \rightarrow f_{ij} - c * \eta_{ij} * g \quad (3)$$

where  $c$  is a constant.

Now, we present the computer simulation procedure of this model in detail:

- 1) Variable initialization. In the 2-dimensional  $n \times n$  neural network model, let the initiatory state and local field equal 0; random initialize each connection weight among  $[-1, 1]$ ; and produce  $M$  random input patterns,  $\zeta_j^i \in [-1, 1]$ ,  $(i = 1, 2, \dots, M; j = 1, 2, \dots, h)$ .
- 2) Learning process. According to formula (1), we input the pattern circularly and iterate the neuron state and local field. After period of time, the space state of the network reaches stability and we consider the  $M$  input patterns have been stored.
- 3) Associative memory. Input a new pattern, and then search the unstable neuron  $(i, j)$  as defined above in whole neural network. Due to being unstable, the neuron  $(i, j)$  discharges a pulse to the each nearest neighbor  $(i', j')$  and thus causes the local fields of them to change as following:

$$f_{i'j'} \rightarrow f_{i'j'} - \frac{\gamma}{2} \eta_{i'j'} (1 + |f_{ij}|) \quad (4)$$

where  $\gamma$  represents the pulse intensity, symbol  $||$  denotes absolute value.

Simultaneously, according to formula (1), the neuron  $(i, j)$  becomes stable as:  $\eta_{ij} \rightarrow \sigma(f_{ij})$ ,  $f_{ij} \rightarrow \eta_{ij}$ ; where  $\sigma(x)$  is sign function.

Repeat this procedure until all neurons of the model are stable. Define one avalanche as all unstable neurons in this process. Then begin drive process by formula (3) and new avalanche.

### III. SIMULATION RESULTS

Recently, Bak and Sneppen have investigated the power law distribution  $P(X)$  of the distances  $X$  between subsequent unstable sites in lattice of BS biology evolution model [3], and J.De.Boer et al. have found some interested result with it [9]. So in this paper we studied not only the distribution  $P(S)$  of the avalanche sizes  $S$  but also the distribution  $P(X)$  of the distances  $X$  between the subsequent unstable sites. We find the distribution will deviate from the power law in some conditions.

#### A. The effect of synchronized area on SOC behavior

The size of our lattice is  $40 \times 40$ . We find that in associative memory process, the distribution of the avalanche sizes has power-law behavior,  $P(S) \propto S^{-\tau}$ ,  $\tau \approx 0.90$ . It is shown in Fig.1.a. The distribution  $P(X)$  of the distances  $X$  between the subsequent unstable sites has power-law behavior too,  $P(X) \propto X^{-\beta}$ ,  $\beta \approx 2.23$ , it is shown in Fig.1.b.

The SOC behavior changed with the scope of lateral connection has been studied. We increase the excitatory lateral connection radius  $d_e$  and the inhibitory connection radius  $d_i = 3d_e$ . It can be seen in Fig 1.a that avalanche size and occurring probability of large scale avalanche decrease with the slope  $d_e$  increasing, and the distribution of  $P(X)$  deviates from power law more and more in large  $X$ , the probability of large distance  $X$  between the subsequent unstable sites also increases, we consider it is a deviation from SOC behavior, it is shown in Fig 1.b.

By investigating, we think our system is in a "partly-synchronized" state, hence the dynamics behavior mentioned above can be seen.

A.Corral et al. propose SOC state and synchronization state might be considered as two uttermost state of system (just like two sides of the same coin) [4]. The inhomogeneity introduced by boundary or initialization conditions can propagate into interior of network, hence makes the system evolve into SOC state [10, 11]. When inhomogeneity is not large enough, the system finds a compromise between synchronization and SOC [6]. It can be considered as a partly-synchronized state.

If Our Model is only a pure OFC model without learning process, the system will present a macroscopic SOC behavior among almost all the lattices, but it is also a neuron network

model. As a kind of self-organized feature map model, after learning a while, it's neurons will develop a unique lateral interaction "Mexican hat" profile that represents its long-term associations with each other. the afferent input weights will self-organize into a topological map of the input space [8], it can form some special topological feature regions. We consider that in these regions, as the connection weights become more topographically ordered, neuron's synchronization effect between each other will be reinforced. When order in these regions is applied to the model, the system has a tendency from SOC state to synchronized state. At last, the system finds a compromise between synchronization and SOC, it could be seen as "partly-synchronized" state. With the process, the distribution of avalanche varies from the continuous distribution to a discrete one, the possibility of large scale avalanche propagating into the interior of the synchronization regions will reduce, and occurring probability of large scale avalanche will decrease too, the one-off isolated avalanche(only one unstable site in an avalanche)in these regions will increase greatly, it makes the distances  $X$  between the subsequent unstable sites have a stochastic spatial even distribution in the area. This distribution has more effect on probability of large distance between unstable sites than probability of small distance. (Because probability of small distance is larger than one of large distance.)

So at this moment, there are some areas in synchronized state and another regions in SOC state. We can consider that with the increasing of  $d_e$ , the feature region(synchronized region) produced by self-organized process becomes wider, which results in the whole system dynamics deviating largely from SOC state, it can be seen in Fig.1.

To verify the idea mentioned above, we draw the avalanche's distribution map of the whole system. We draw one avalanche's distribution snapshot every 1000 avalanches, then overlap all the snapshots in one picture. The result can be seen in Fig2. It can be clearly seen that the blank region (seldom avalanche area) expands with the increasing of  $d_e$ . It means that the synchronized region introduced by self-organized expands and large scale avalanches reduce more and more. Even though, there are still some isolated unstable neurons in the area, it indirectly verifies our deduction of  $p(X)$  distribution mentioned above.

We investigate the relation between average avalanche size  $\langle S \rangle$  and radius  $d_e$ . From Fig.3 , we can see that with the decreasing of  $d_e$  ,  $\langle S \rangle$  will increases, and with  $d_e$  approaching 0, the slope of the curve becomes quite large. This result is approximate to the phenomena with the increasing of pulse discharging intensity  $\gamma$  [7], and is consistent with the result in

Ref. [12]. It implies that the network approaches to SOC state when  $d_e$  approaching 0 or  $\gamma$  approaching 0.5.

When the ratio  $T$  of radius  $d_i$  with radius  $d_e$  is changed, the probability distribution  $P(S)$  and  $P(X)$  and avalanche distribution are changed too, the tendency is similar to Fig.1, Fig.2. With the increment of  $T$ , the partly-synchronized behavior of system becomes distinctness. The phenomena can also be explained by the expanding of synchronized region, but it leads us to think that the inhibitory lateral connection and excitatory lateral connection have what different effect on synchronized process? It is look like that the inhibitory lateral connection has more important effect, but it is not very clear, there are still a lot of work to do.

## B. The approximate behavior in a kind of quasi-OFC earthquake model

OFC earthquake model is a kind of SOC model which has been widely studied in recent years [2]. If we use transformation in our model :  $1 - \eta_{ij} f_{ij} \rightarrow F_{ij}$ , then formula (4) become  $F_{i'j'} \rightarrow F_{i'j'} + \frac{\gamma}{2} F_{ij}$ . It is the avalanche mechanism of the OFC earthquake model, in fact these two models belong to the same class. Therefore we add some synchronized regions in OFC model and examine that if the system has the similarity partly-synchronized behavior as in our neuron network model. Hence we introduced a cellular automaton model based on OFC model, the steps are as follows:

- 1) We define a  $N \times N$  square lattice, and a  $L \times L$  square area in the lattice ( $L = KN < N$ )
- 2) Initialize all sites to a random value  $F_{ij}$  between 0 and 1.
- 3) If any  $F_{ij} \geq F_{th} = 1$ , then redistribute the force on  $F_{ij}$  to its neighbors according to the rule:

$$F_{i'j'} \rightarrow F_{i'j'} + \frac{\gamma}{2} F_{ij} \quad \begin{cases} \gamma = \alpha < 0.5, & \text{when site } (i, j) \in L \times L \text{ region,} \\ \gamma = 0.5 & \text{otherwise.} \end{cases} \quad (5)$$

$$F_{ij} \rightarrow 0$$

- 4) Repeat step 3 until no  $F_{ij} \geq F_{th}$ , we define the avalanche is fully evolved.
- 5) Locate the site with the largest strain,  $F_{max}$ , then drive all sites

$$F_{ij} \rightarrow F_{ij} + (F_{th} - F_{max}) \quad (6)$$

and return to step 3. Here we use open boundary conditions ,and serial working mode. It is same as our neuron network model but different from traditional OFC model.

We still focus on the spatial distribution of  $S$  and  $X$ . Let  $\alpha = 0.1$ , then change  $K$ , the result is shown in Fig 4. It can be seen that the probability distribution  $P(X)$  increases and deviates from power-law more with  $K$  increasing at larger  $X$  in Fig 4.a. The tendency is similar to that in our previous neuron network model. In Fig 4.b , we can see the occurring probability of large scale avalanche reduces with  $K$  increasing too, but the phenomena is not distinctness as that in previous model. The reason may be that the previous model has more complex structure than this model. It produces many pieces of synchronized region, but here, we only add a piece of synchronized region in model.

A. Corral, Grassberger et al. have investigated the influence of pulse discharging intensity  $\gamma$  in OFC model. They propose the model present a macroscopic synchronization among all the elements of the lattice when  $\gamma$  is small [4, 13]. Therefore in this model , with expanding of  $L \times L$  area( $\gamma = \alpha = 0.1$ ), the synchronized area in the network expands, the synchronization behavior becomes distinctness, and the whole system evolves into a partly-synchronized state. The dynamics behavior of the previous model is so consistent with this model, it suggested that maybe they have the similar dynamics mechanism . Our results tend to agree with this idea.

For studying the partly-synchronized phenomena further, the relation between distribution  $P(x)$  and  $\alpha$  has been investigated. Let  $K = 0.8$  , the partly-synchronized behavior become distinctness with  $\alpha$  decreasing from 0.5 . But when  $\alpha$  less than a number (about 0.4), there is no more distinctness partly-synchronized behavior introduced by  $\alpha$  changing (seen in Fig 5) . It can show that the  $L \times L$  area have evolved into synchronized state. We also draw the whole network's avalanches distribution map(Fig .6). It can be clearly seen that the isolated avalanches' number in  $L \times L$  area increases with  $\alpha$  decreasing, and the large scale avalanche can't propagate into the interior of  $L \times L$  area. These result are consistent with the work of C.Tang , Grassberger et al. [10, 13].

#### IV. CONCLUSION

In this paper, we analyze the dynamics of the proposed neural network model and find the distribution of the avalanche sizes and the distances  $X$  between subsequent unstable

sites show the power-law behavior. More important, we find the function area and weight distribution produced by self-organized process in our Neural Network model will let the  $P(S)$  and  $P(X)$  distribution deviate from power law behavior , and the system evolves into a partly-synchronized state at this time. To verify the explanation , we study a quasi-OFC earthquake model containing synchronized region , and find it will deviate from power-law , evolve into a partly-synchronized state in some conditions too.

Our self-organized feature map Neuron Network model is just a very simple simulation of brain. The real brain has very complex structure and more specific feature regions in Cortex. So brain may express a quasi-SOC(partly-synchronized) behavior more than a pure SOC behavior. Therefore the stored pattern in brain might be designed as a quasi-SOC attractor, and the associate memory process might be designed as the process of the input pattern evolving into the attractor.

Now the neuron synchronization in brain has been observed in many experiments [14]. Rodriguez et al. have investigated the long distance synchronization of human brain activity [15]. We think it would be interesting to further investigate the relationship between synchronization and cognitive , associate process.

---

- [1] P. Bak, C. Tang, and K. Wiesenfield, Phys. Rev. A 38 (1988) 364
- [2] Z. Olami, S. Feder, and K. Christensen, Phys. Rev. Lett. 68 1244 (1992)
- [3] P. Bak, and K. Sneppen, Phys. Rev. Lett. 71 , 4083(1993)
- [4] A. Corral, et al., Phys. Rev. Lett. 74 , 118(1995)
- [5] N. Mousseau, Phys. Rev. Lett. 77 , 968(1996)
- [6] S. Bottani, Phys. Rev. Lett. 74 , 4189(1995)
- [7] Huang Jing, and Chen TianLun to be published in Comm. Theor. Phys
- [8] J. Sirosh, and R. Miikkulainen, Biol. Cybern. 71 , 65(1994)
- [9] J. De. Boer et al. Phys. Rev. E 51 , 1059(1995)
- [10] A. A. Middleton, and C. Tang Phys. Rev. Lett. 74 , 742(1995)
- [11] H. Ceva, Phys. Lett. A 245 , 413(1998)
- [12] M. L. Chabanol, and V. Hakim, Phys. Rev. E 56 , R2343(1997)
- [13] P. Grassberger, Phys. Rev. E. 49 , 2436(1994)
- [14] Singer W. Annul. Rev. Physiol. 55 , 359(1993);  
Singer W. and Gray C. M. Annul. Rev. Neurosci. 18 , 555(1995) ;  
Nelson J. L. et al. Vis. Neurosci 9 , 21(1992)
- [15] Eugenio Rodriguez, et al. Nature 397(1999)

FIG. 1: The distribution  $P(S)$  of the avalanche sizes  $S$ , and the distribution  $P(X)$  of distances  $X$  between subsequent unstable sites, changed with lateral connection radius  $d_e$ , for our  $40 \times 40$  neuron network model. Where  $d_i = 2d_e$ ,  $\gamma = 0.5$ . (a). Log-log plot of  $P(S)$  .vs. sizes  $S$ . (b). Log-log plot of  $P(X)$  .vs. distances  $X$ .

FIG. 2: The avalanche distribution map for our  $40 \times 40$  neuron network model , where  $d_i = 2d_e$ ,  $\gamma = 0.5$ . (o) represent the origin sites of avalanche. shadow(+) represent the avalanche region. (a)-(d) correspond to  $d_e = 1, 2, 4, 6$

FIG. 4: The distribution  $P(X)$  of the distance  $X$  between subsequent unstable sites and the distribution  $P(S)$  of the avalanche sizes  $S$  , changed with  $K = L/N$ . for our  $40 \times 40$  quasi-OFC earthquake model. Where in  $L \times L$  area, $\gamma = \alpha = 0.1$ , out of this area,  $\gamma = 0.5$ . (a). Log-log plot of  $P(X)$  .vs. distances  $X$  . (b). Log-log plot of  $P(S)$  .vs. sizes  $S$ .

FIG. 5: The distribution  $P(X)$  of the distance  $X$  between subsequent unstable sites and the distribution  $P(S)$  of the avalanche sizes  $S$  , changed with pulse discharging intensity  $\alpha$  in  $L \times L$  area for our  $40 \times 40$  quasi-OFC earthquake model. Where  $K = L/N = 0.8$ , out of  $L \times L$  area  $\gamma = 0.5$ . (a). Log-log plot of  $P(X)$  .vs. distances  $X$ . (b). Log-log plot of  $P(S)$  .vs. sizes  $S$ .

FIG. 6: The avalanche distribution map for our  $40 \times 40$  quasi-OFC earthquake model , where  $K = L/N = 0.8$ , out of  $L \times L$  area  $\gamma = 0.5$ . pulse discharging intensity  $\alpha$  is changed. (o) represent the origin sites of avalanche. shadow(+) represent the avalanche region. The map(a)-(d) correspond to  $\alpha = 0.5, 0.48, 0.4, 0.1$

FIG. 3: The avalanche average size  $\langle S \rangle$  as a function of the lateral connection radius  $d_e$  for our  $40 \times 40$  neuron network model , where  $d_i = 2d_e$ ,  $\gamma = 0.5$ . With decreasing of  $d_e$ ,  $\langle S \rangle$  increases, and with  $d_e$  approaching 0, the slope of the curve become quite large.



























